Indian Statistical Institute
First Semester 2004-2005
Mid-Semestral Exam, B.Math (Hons.) I Year
Analysis I

Time: 3 hrs   Date:23-09-04

1. On the set $\mathbb{C}$ of complex numbers define the lexicographic order as follows: For $z = a + bi, w = c + di$, with $a, b, c, d \in \mathbb{R}$, define $z < w$ if $a < c$, and also if $a = c$ but $b < d$. Prove that this turns $\mathbb{C}$ into an ordered set. Show that $\mathbb{C}$ under this order does not have the least upper bound property. [20]

2. Let A be the collection of all finite subsets if the set of natural numbers $\mathbb{N}$. Show that A is countable. [20]

3. Find $\limsup$ and $\liminf$ of the following sequences: (i) $a_n = (-1)^n + \frac{1}{n}$ $(ii) b_n = (-1)^n (4 + \frac{5}{n}$ $(iii) d_n = \frac{(n + (-1)^n (2n+1))}{n}$, $n \geq 1$. [15]